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Anomaly matching & S^1 compactification

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1710.08923 w./ T. Misumi & N. Sakai.

· Anomaly matching

QFT with sym. $\nrightarrow G \Rightarrow A$: G-gauge field

$$Z[A+d\theta] = Z[A]\, e^{i\mathcal{A}(\theta,A)}$$

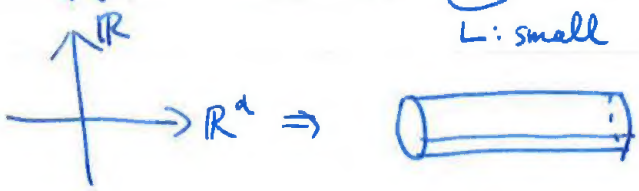
$\mathcal{A}$: anomaly. $\Rightarrow$ RG-inv.

✓ Vacua must reproduce $\mathcal{A}$. ('t Hooft anomaly matching).

· Volume indep. (or. Adiabatic compactification)

$\mathbb{R}^{d+1} \Rightarrow \mathbb{R}^d \times S^1$

L: small Weak coupling (Asymptotic freedom)



✓ Claim: Vacuum of $\mathbb{R}^{d+1}$ = Vacuum of $\mathbb{R}^d \times S^1$.

with appropriate +.b.c.

To be consistent,

Anomaly on $\mathbb{R}^d \times S^1$ = Anomaly on $\mathbb{R}^{d+1}$.

Q. Is it possible?

· Difficulty : Thermal fluc. eliminates $\mathcal{A}$ usually.

ex. 3D free Dirac fermion : $U(1)$ & T.

$\mathcal{A}$.

$$Z[T\cdot A] = Z[A]\, \exp\left(-\tfrac{i}{4\pi}\int A dA\right).$$

Set $M^3 = M^2 \times S^1_L$ & $L \ll \mathrm{size}(M^2)$.

A_2: $U(1)$-conn. on $M^2 \Rightarrow A_2 dA_2 = 0$. No anomaly.

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Solve this difficulty.

- Pure YM. (Gaiotto, Kapustin, Komargodski, Seiberg)
  or Theories with 1-form symm.

- Theories without 1-form symm. (our work)
  e.g. $\mathbb{CP}^{N-1}$ model
  $\Longrightarrow$ Massless N-flavor QCD with flavor twisted b.c.

- Pure SU(N) YM @ $\theta=\pi$.

Symm: $\mathbb{Z}_N$ one-form & T.

$W(C)=\mathrm{tr}(\mathcal{P}e^{i\int a})$.

$\mapsto \omega\, W(C)$. $(\omega=e^{2\pi i/N})$

$\mathbb{Z}_N$ two-form gauge field B: $\mathcal{Z}[B]$

$$\mathcal{Z}[T\cdot B] = \mathcal{Z}[B]\, e^{\frac{iN}{4\pi}\int B\wedge B}$$

$\mathbb{R}^4 \Rightarrow \mathbb{R}^3\times S^1$.

Polyakov loop $\mathrm{tr}(\mathcal{P}e^{i\int_0^L a})$.

$\mathbb{Z}_N$ 1-form $\Rightarrow$ $\mathbb{Z}_N$ 0-form. $B^{(1)}$ + $\mathbb{Z}_N$ 1-form $B^{(2)}$.

Anomaly in 3D: $B\equiv B^{(2)} + B^{(1)}\wedge L^{-1}dx^4$,

$$\mathcal{Z}[T\cdot(B^{(1)},B^{(2)})] = \mathcal{Z}[(B^{(1)},B^{(2)})]\, e^{\frac{iN}{2\pi}\int B^{(1)}\wedge B^{(2)}}.$$

$\Rightarrow$ Good for V.I.

We can claim that both vacua are controlled by the same anomaly matching.

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- $\mathbb{C}P^{N-1}$. (Dunne, Ünsal, ..., Sulejmanpasic)

$$S = S\left(|(\partial+ia)\vec{z}|^2 + V(|\vec{z}|^2)\right) + \frac{i\theta}{2\pi}\int da.$$

Symm: $\begin{cases} \text{flavor } \frac{SU(N)}{\mathbb{Z}_N}, & \vec{z} \mapsto U\cdot\vec{z} \\ \text{(x) } \theta=\pi, & T. \end{cases}$

$\mathbb{R}^2 \Rightarrow \mathbb{R}\times S^1$.

$\vec{z}(x^1, x^2+L) = \Omega\cdot\vec{z}(x^1, x^2)$. $\quad \Omega = \begin{pmatrix} 1 & & 0 \\ & \omega & \\ 0 & & \ddots \\ & & & \omega^{N-1} \end{pmatrix} \in SU(N)/\mathbb{Z}_N$.

$\Rightarrow$ Large-N V.I.

- Anomaly.

flavor gauge field

A - $SU(N)$, $\quad B$ - $\mathbb{Z}_N$.

$$Z_{\theta=\pi}[T\cdot(A,B)] = Z_{\theta=\pi}[(A,B)]\, e^{i\int B}.$$

$\mathbb{R}^2 \Rightarrow \mathbb{R}\times S^1$.

$\Omega = \mathcal{P} e^{i\oint_0^L A}$: Background $SU(N)$ holonomy.

$\mathbb{Z}_N$ 1-form : $\Omega \to \omega\cdot\Omega$.

B.C. is changed $\Rightarrow$ Different theories.

Shift symm.

$$\vec{z} = \begin{pmatrix} z_1 \\ \vdots \\ z_N \end{pmatrix} \longmapsto \begin{pmatrix} z_2 \\ \vdots \\ z_N \\ z_1 \end{pmatrix} \equiv S\cdot\vec{z}.$$

This also changes B.C.:

$$\Omega \to S\Omega S^{-1} = \omega\Omega.$$

Intertwined $\mathbb{Z}_N$ symm: $-B^{(1)}$.

$\Omega \xrightarrow{\text{shift}} S\cdot\Omega\cdot S^{-1} \xrightarrow{\mathbb{Z}_N\text{-0-form}} \omega^{-1}\cdot S\cdot\Omega\cdot S^{-1} = \Omega$.

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$B = B^{(1)} \wedge L^{-1} dx^2$: Acts on Polyakov loop $\in U(1)$

$e^{i\oint_0^L a} \to \omega\, e^{i\oint_0^L a}$.

$Z_\Omega[T\cdot(A,B)] = Z_\Omega[(A,B)]\, e^{i\int B^{(1)}}$.

Thm (Y.T.-Misumi-Sulkai)

QFT with symm. $G = \frac{SU(N)}{\mathbb{Z}_N}$ (A) & $H \ni h$ (B)

Anomaly

$Z[h\cdot(A,B)] = Z[(A,B)]\, e^{i\mathcal{A}_h[B]}$.

$\mathbb{R}^{D+1} \Rightarrow \mathbb{R}^D \times S^1$. w/ Background $SU(N)$ hol. $\Omega = \mathcal{P}e^{i\int A}$

s.t. $\exists S \in SU(N)$ $S\cdot\Omega\cdot S^{-1} = \omega\cdot\Omega$.

$\Rightarrow$ Mixed anomaly among $(\mathbb{Z}_N)_{\text{shift}}$ ($B^{(1)}$), $\frac{Ab(SU(N))}{\mathbb{Z}_N}$ (A', $B^{(2)}$), H_h

$Z[h\cdot(A', B^{(1)}, B^{(2)})]$

$= Z[(A', B^{(1)}, B^{(2)})]$

$\times \exp\left(i\mathcal{A}_h[B^{(2)} + B^{(1)} \wedge L^{-1} dx^{D+1}]\right)$. //

Application Massless N-flavor QCD ($N_c = N_f = N$)

(Related works: Shimizu, Yonekura; Gaiotto, Kapustin, Seiberg)

Symm. $SU(N)/\mathbb{Z}_N$ (A, B), $(\mathbb{Z}_{2N})_{\text{axial}}$.

$Z[(A,B)] \xrightarrow{(\mathbb{Z}_{2N})_{\text{axial}}} Z[(A,B)]\, e^{\frac{2iN}{4\pi}\int B\wedge B}$.

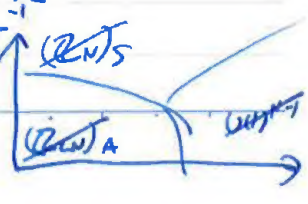
anomaly if $N \geq 3$.

Flavor twisted b.c.

$q_n(x, x^4 + L) = q_n(x, x^4)\, e^{\frac{2\pi i n}{N}}$ $(n = 0, \cdots, N-1)$ (u, d, s)

$q_i \to q_{i+1}$, Polyakov $\to \omega\cdot$Polyakov

Symm: $(\mathbb{Z}_N)_{\text{shift}}$ ($B^{(1)}$), $\frac{U(1)^{N-1}}{\mathbb{Z}_N}$ (A', $B^{(2)}$), $(\mathbb{Z}_{2N})_{\text{axial}}$.

$Z[(A', B^{(1)}, B^{(2)})] \xrightarrow{(\mathbb{Z}_{2N})_{\text{axial}}} Z[(A', B^{(1)}, B^{(2)})]\, e^{\frac{2iN}{2\pi}\int B^{(1)}\wedge B^{(2)}}$.

No trivially gapped phase for $\mathbb{Z}_N$-QCD!!